ÇUBUQ TİPLİ KONSTRUKSİYA ELEMENTİNİN DARTILMADA ZƏDƏLƏNMƏSİNİN RİYAZİ MODELLEŞDİRİLMƏSİ

S.A.PİRİYEV, G.B.HÜSEYNOVA

XÜLASƏ

Müasir konstruksiya texnologiyalarının inkişafı, onun elementlərinin hərtərəfli hesablanmasının düzgünlüyünü və habelə onların istismar müddətinin qiymətləndirilməsinə artan tələblər qoyur. Mürəkkəb gərginlik vəziyyətində olan materialların və konstruksiyaların davranışının öyrənilməsi bərk cisim mexanikasının inkişafının ən vacib istiqamətlərindən biri hesab edilir. Yüklənmə zamanı deformasiya prosesi ilə birlikdə, dağılmanı zamana bağlı proses kimi qəbul edərək, konstruksiya materialının həcmində müxtəlif növ defektlərin əmələ gəlməsini və yığılmasını vahid terminlə (zədələnmə) birləşdirilərək zədələnmə nəzəriyyələri yaradılmışdır. Xüsusilə polimer və kompozit materialların texnikada və sənayedə geniş tətbiq edildiyi üçün, onların dağılması dedikdə, fiziki nöqteyi-nəzərdən onların xüsusi möhkəmliyinin azalması, özlüelastiki dağılması və müvafiq olaraq çəkisinin azaması kimi başa düşülür. Eksperimental diaqramların analizi göstərir ki, bu materialların sürüncəklik deformasiyası həmişə geridönməzdir və qalıq deformasiya yüklənmə zamanı yığılmış defektlərin həcmi ilə müəyyən edilir.

Məqalədə, çubuq tipli konstruksiya elementlərinin dartılması zamanı zədələrin toplanmasının kinetikasına baxılmışdır. Dağılma mexanikasının prinsiplərinə əsaslanaraq, bu xüsusiyyətlər nəzərə alınmaqla zədələnmənin toplanması kinetikasının bəzi sadə riyazi modelləri nəzərdən keçirilir.

Açar söz: zədələnmə, dağılma, gərginlik, sürüncəklik, deformasiya.

ON MATHEMATICAL MODELING OF THE DAMAGE OF ROD-TYPE STRUCTURAL ELEMENTS UNDER STRETCHING

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SUMMARY

Modern technology imposes increased requirements on the accuracy and thoroughness of the calculation of structural elements and the structures themselves, as well as on the assessment of their working life. The study of the behavior of materials and structures under conditions of a complex stress state is one of the most important directions in the development of solid mechanics. Understanding destruction as a temporary process, in conjunction with the process of deformation, led to the emergence of theories of damage, when various kinds of defects are formed and accumulate in the volume of the material of the structure during loading, united by a single term - damage. Such a physical picture of the development of the fracture process is especially evident for polymer and composite materials, the widespread use of which in engineering and industry is explained by their high specific strength and fracture toughness, and, accordingly, the possibility of reducing the weight of parts and structures. Experimental diagrams show that the creep deformation of these materials is not always reversible - the residual deformation is determined by the volume of defects accumulated during loading.

The article deals with the issue of the kinetics of damage accumulation in rod-type structural elements in tension. Based on the principles of fracture mechanics, some simple mathematical models of the kinetics of damage accumulation are considered, taking into account these features.

Key words: damageability, fracture, stress, creep, deformation.

İNFORMATİKA

Mathematics Subject Classification: **60K15 + 44A10 + 68N15**

INVESTIGATION OF THE CONDITIONAL DISTRIBUTION OF A SEMI-MARKOV RANDOM WALK PROCESS USING THE MAPLE SOFTWARE PACKAGE

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This paper considers the sequence $\{\xi_k^+, \eta_k^+, \xi_k^-\}_{k=1}^{\infty}$ *of independent identically distributed, positive, independent random variables and the sequence* $\{\eta_k^-\}_{k=1}^{\infty}$ *of negative random variables. On the basis of these random variables, a semi-Markov random walk process with a delaying screen at zero is constructed, and an integral equation for the conditional distribution* $R(t, x|z, h)$ *of this process is found using the formula of total probability. In the class of distributions decreasing exponentially fast, using the method of successive Laplace integral transforms in time t and Laplace-Stiltes in phase x, this integral equation is reduced to a partial differential equation – to the fourth-order Mangeron equation. The resulting differential equation is solved in the Maple package for some special cases and 3D graphic images are obtained.*

Keywords: Mangeron equations**,** Laplace-Stieltjes transform, independent random variables, Semi-Markov random walk, Maple package

1. Introduction

To study the distribution of a semi-Markov random walk and its main boundary functionals, some authors used asymptotic, factorization, and other methods [3-7]. In this paper, narrowing the class of the random walk, the integral equation for the Laplace transform in time, the Laplace-Stieltjes transform in the phase of the conditional distribution of the semi-Markov random walk process are reduced to the Mangeron equation [1-2, 8, 11-12]. Note that many problems of mathematical physics related to the phenomena of vibration, and problems of automatic control, in which it is necessary to take into account the dependencies not only between velocities, but also accelerations and even higher derivatives, are reduced to the Mangeron equations. The resulting differential equation is solved in the Maple package for some special cases and 3D graphic images are obtained.

2. Probabilistic problem statement

Suppose the sequence $\{\xi_\kappa^+, \eta_k^+, \xi_\kappa^-\}_{k=\overline{1,\infty}}$ is given on the probability space $(\Omega, F, P(\cdot))$ of independent identically distributed, positive, independent random variables and the negative random variable $\eta_k^- < 0. k = \overline{1,\infty}$.

Using these random variables, we construct the following random processes

$$X^\pm(t) = \sum_{i=1}^{k-1} \eta_i^\pm, \text{ if } \sum_{i=1}^{k-1} \xi_i^\pm \le t < \sum_{i=1}^{k} \xi_i^\pm, \; k = \overline{1,\infty}.$$

We can write these processes in the following form:

$$X^\pm(t) = \sum_{i=1}^{\vartheta^\pm(t)} \eta_i^\pm, \text{ where } v^\pm(t) = k, \text{if} \sum_{i=1}^{k-1} \xi_i^\pm \le t < \sum_{i=1}^{k} \xi_i^\pm.$$

Let us call the process $X_1(t) = X^+(t) - X^-(t)$ a complex semi-Markov random walk process.

Denote

$$\tau_k^\pm = \sum_{i=1}^{k} \xi_i^\pm, \quad k = \overline{1,\infty}$$

We arrange these random variables in ascending order

$$\{\tau_k\}, \; k = \overline{1,\infty}.$$

Denote

$$\eta_k = \begin{cases} \eta_i^+, & \text{if } \tau_k = \tau_i^+, \\ \eta_j^-, & \text{if } \tau_k = \tau_j^-. \end{cases}$$

Construct the following process

$$X(t) = \varsigma_{k,} \text{ if } \tau_k \le t < \tau_{k+1}, \quad k = \overline{1,\infty},$$
$$\varsigma_0 = z, \quad \varsigma_k = max(0, \varsigma_{k-1} + \eta_k), \; z > 0$$

Let us call the process $X(t)$ differential with a random walk and with a delaying screen at zero. The aim of this paper is to study the distribution of this semi-Markov process.

3. Solution

Note that neither the moments τ_k^+,, nor the moments τ_k^- are Markov moments. If we know the value of the process $X(t)$ at the moment t, then to determine the further behavior of the process, we also need to know when a positive jump will occur for the first time after τ_k^-. Therefore, when studying the process $X(t)$, it is natural to also consider two following processes.

$$\delta^\pm(t) = min[\tau_k^\pm - t]$$

We must investigate the distribution of the process $X(t)$ in the following form

$$P\{X(t) < x | X(0) = z, \delta^+(0) = h\} = P\{X(t) < x | X(0) = z, \xi_1^+ = h\}$$

or

$$P\{X(t) < x | X(0) = z, \delta^-(0) = h\} = P\{X(t) < x | X(0) = z, \xi_1^- = h\}.$$

Denote

$$R(t, x|z, h) = P\{X(t) < x | X(0) = z; \xi_1^+ = h\}.$$

In the case where the random variables have an exponential distribution with the parameters $\lambda_{\pm}$ and $\mu_{\pm}$, respectively, for the double integral image of the conditional distribution $R(t,x|z,h)$ in [1], the 4th-order Mangeron equation was obtained:

$$\frac{\partial^4\tilde{\tilde{R}}(\theta,\alpha|z,h)}{\partial z^2\partial h^2}+2(\lambda_-+\theta)\frac{\partial^3\tilde{\tilde{R}}(\theta,\alpha|z,h)}{\partial z^2\partial h}+2\mu_-\frac{\partial^3\tilde{\tilde{R}}(\theta,\alpha|z,h)}{\partial z\partial h^2}+$$
$$+4(\lambda_-+\theta)\mu_-\frac{\partial^2\tilde{\tilde{R}}(\theta,\alpha|z,h)}{\partial z\partial h}+\mu_-^2\frac{\partial^2\tilde{\tilde{R}}(\theta,\alpha|z,h)}{\partial h^2}+(\lambda_-+\theta)\frac{\partial^2\tilde{\tilde{R}}(\theta,\alpha|z,h)}{\partial z^2}+$$
$$+2(\lambda_-+\theta)\mu_-^2\frac{\partial\tilde{\tilde{R}}(\theta,\alpha|z,h)}{\partial h}+(\lambda_-+\theta)^2\mu_-\frac{\partial\tilde{\tilde{R}}(\theta,\alpha|z,h)}{\partial z}+[2\lambda_-(\lambda_-+\theta)+\theta^2]\times$$
$$\times\mu_-^2\tilde{\tilde{R}}(\theta,\alpha|z,h)=(\alpha-\mu_-)^2(2\lambda_-+\theta)e^{-\alpha z},$$

where

$$\tilde{\tilde{R}}(\theta,\alpha|z,h)=\int_{x=0}^{\infty}e^{-\alpha x}d_x\left\{\int_{t=0}^{\infty}e^{-\theta t}R(t,x|z,h)dt\right\}.$$

The solution of the resulting equation is found in the environment of the mathematical package MAPLE [9-10]. A program is developed for constructing the solution surface of the corresponding Cauchy problem for the case when the random variables $\boldsymbol{\xi_i}$ and $\boldsymbol{\eta_i}$ have an exponential distribution with the parameter equal to one.

```
> restart;
> with(inttrans):assume(x>0,t>0,z>0,h>0);with(plots):
> mang:=diff(K(z,h),z$2,h$2)+a1*diff(K(z,h),z$2,h)+a2*diff(K(z,h),
z,h$2)+a3*diff(K(z,h),z,h)+a4*diff(K(z,h),h$2)+a5*diff(K(z,h),z$2)+a6*diff
(K(z,h),h)+a7*diff(K(z,h),z)+a8*K(z,h)=a9*exp(-alpha*z);
```

$$mang := \frac{\partial^4}{\partial h\sim^2\partial z\sim^2}K(z\sim,h\sim)+a1\left(\frac{\partial^3}{\partial h\sim\partial z\sim^2}K(z\sim,h\sim)\right)+a2\left(\frac{\partial^3}{\partial h\sim^2\partial z\sim}K(z\sim,h\sim)\right)$$
$$+a3\left(\frac{\partial^2}{\partial h\sim\partial z\sim}K(z\sim,h\sim)\right)+a4\left(\frac{\partial^2}{\partial h\sim^2}K(z\sim,h\sim)\right)+a5\left(\frac{\partial^2}{\partial z\sim^2}K(z\sim,h\sim)\right)$$
$$+a6\left(\frac{\partial}{\partial h\sim}K(z\sim,h\sim)\right)+a7\left(\frac{\partial}{\partial z\sim}K(z\sim,h\sim)\right)+a8\,K(z\sim,h\sim)=a9\,e^{-\alpha z\sim}$$

According to the form of the inhomogeneous part of equation (1), its solution is sought in the form *K(z,h)=f(h)·exp(-α·z)*

> eq:=subs(K(z,h)=f(h)*exp(-alpha*z),mang);

$$eq := \frac{\partial^4}{\partial h\sim^2\partial z\sim^2}\left(f(h\sim)\,e^{-\alpha z\sim}\right)+(2\lambda+2\theta)\left(\frac{\partial^3}{\partial h\sim\partial z\sim^2}\left(f(h\sim)\,e^{-\alpha z\sim}\right)\right)$$
$$+2\mu\left(\frac{\partial^3}{\partial h\sim^2\partial z\sim}\left(f(h\sim)\,e^{-\alpha z\sim}\right)\right)+4(\lambda+\theta)\mu\left(\frac{\partial^2}{\partial h\sim\partial z\sim}\left(f(h\sim)\,e^{-\alpha z\sim}\right)\right)$$
$$+\mu^2\left(\frac{\partial^2}{\partial h\sim^2}\left(f(h\sim)\,e^{-\alpha z\sim}\right)\right)+(\lambda+\theta)\left(\frac{\partial^2}{\partial z\sim^2}\left(f(h\sim)\,e^{-\alpha z\sim}\right)\right)+2(\lambda$$
$$+\theta)\mu^2\left(\frac{\partial}{\partial h\sim}\left(f(h\sim)\,e^{-\alpha z\sim}\right)\right)+(\lambda+\theta)^2\mu\left(\frac{\partial}{\partial z\sim}\left(f(h\sim)\,e^{-\alpha z\sim}\right)\right)+\left(2\lambda(\lambda\right.$$
$$\left.+\theta)+\theta^2\right)\mu^2 f(h\sim)\,e^{-\alpha z\sim}=(\alpha-\mu)^2(2\lambda+\theta)\,e^{-\alpha z\sim}$$

with respect to *A f(h)* the following equation is obtained

> expand(eq/exp(-alpha*z));

$$\left(\frac{d^2}{dh\sim^2} f(h\sim)\right)\alpha^2 + 2\left(\frac{d}{dh\sim} f(h\sim)\right)\alpha^2\lambda + 2\left(\frac{d}{dh\sim} f(h\sim)\right)\alpha^2\theta - 2\left(\frac{d^2}{dh\sim^2} f(h\sim)\right)\alpha\mu - 4\left(\frac{d}{dh\sim} f(h\sim)\right)\alpha\lambda\mu - 4\left(\frac{d}{dh\sim} f(h\sim)\right)\alpha\mu\theta + \left(\frac{d^2}{dh\sim^2} f(h\sim)\right)\mu^2 + f(h\sim)\alpha^2\lambda + f(h\sim)\alpha^2\theta + 2\left(\frac{d}{dh\sim} f(h\sim)\right)\lambda\mu^2 + 2\left(\frac{d}{dh\sim} f(h\sim)\right)\mu^2\theta - f(h\sim)\alpha\lambda^2\mu - 2f(h\sim)\alpha\lambda\mu\theta - f(h\sim)\alpha\mu\theta^2 + 2f(h\sim)\lambda^2\mu^2 + 2f(h\sim)\lambda\mu^2\theta + f(h\sim)\mu^2\theta^2 = 2\alpha^2\lambda + \alpha^2\theta - 4\alpha\mu\lambda - 2\alpha\mu\theta + 2\mu^2\lambda + \mu^2\theta$$

> eqs:=simplify(%);

$$eqs := (-\alpha+\mu)^2\left(\frac{d^2}{dh\sim^2} f(h\sim)\right) + 2(-\alpha+\mu)^2(\lambda+\theta)\left(\frac{d}{dh\sim} f(h\sim)\right) + \left((2\lambda^2 + 2\lambda\theta + \theta^2)\mu^2 - (\lambda+\theta)^2\mu\alpha + (\lambda+\theta)\alpha^2\right) f(h\sim) = (-\alpha+\mu)^2(2\lambda+\theta)$$

We accept designations

> a1:=2*(lambda+theta);

$$a1 := 2\lambda + 2\theta$$

> a2:=2*mu;

$$a2 := 2\mu$$

> a3:=4*(lambda+theta)*mu;

$$a3 := 4(\lambda+\theta)\mu$$

> a4:=mu^2;

$$a4 := \mu^2$$

> a5:=(lambda+theta);

$$a5 := \lambda + \theta$$

> a6:=2*(lambda+theta)*mu^2;

$$a6 := 2(\lambda+\theta)\mu^2$$

> a7:=(lambda+theta)^2*mu;

$$a7 := (\lambda+\theta)^2\mu$$

> a8:=(2*lambda*(lambda+theta)+theta^2)*mu^2;

$$a8 := \left(2\lambda(\lambda+\theta) + \theta^2\right)\mu^2$$

> a9:=(alpha-mu)^2*(2*lambda+theta);

$$a9 := (\alpha-\mu)^2(2\lambda+\theta)$$

Solving the Cauchy problem with conditions *f(0)=1, D(f)(0)=1*

```
> eqss:=(-a2*alpha+alpha^2+a4)*diff(f(h),h$2)+(a1*alpha^2-a3*alpha+a6)*diff(f(h),h)+(a5*alpha^2-a7*alpha+a8)*f(h)=a9;
```

$$eqss := \left(\alpha^2 - 2\,\alpha\,\mu + \mu^2\right)\left(\frac{d^2}{dh\sim^2} f(h\sim)\right) + \left((2\,\lambda + 2\,\theta)\,\alpha^2 - 4\,(\lambda+\theta)\,\mu\,\alpha + 2\,(\lambda + \theta)\,\mu^2\right)\left(\frac{d}{dh\sim} f(h\sim)\right) + \left((\lambda+\theta)\,\alpha^2 - (\lambda+\theta)^2\,\mu\,\alpha + \left(2\,\lambda\,(\lambda+\theta) + \theta^2\right)\mu^2\right) f(h\sim) = (\alpha-\mu)^2\,(2\,\lambda+\theta)$$

```
> ics:=f(0)=1,D(f)(0)=1;
```

$$ics := f(0) = 1,\ \mathrm{D}(f)\,(0) = 1$$

```
> w:=unapply(simplify(subs({mu=1,lambda=1},dsolve({eqss,ics},f(h)))),h,alpha,theta):
```

The solution of the Mangeron equation is obtained in the form

```
> Kzh:=unapply(w(h,alpha,theta)*exp(-alpha*z),z,h,theta,alpha);
```

$$Kzh := (z\sim, h\sim, \theta, \alpha) \mapsto e^{-\alpha\cdot z\sim}\cdot f(h\sim) = \Bigg(e^{-\alpha\cdot z\sim}\cdot\Bigg(\Big(-\big(\big(\theta^2-1\big)\cdot\alpha - \theta^2 - \theta - 1\big)\cdot(\alpha-1)\cdot(2+\theta)\cdot\sqrt{-1+\big(\theta^2+\theta\big)\cdot\alpha^2 - (\theta+1)^2\cdot\alpha} - \big(-1+\big(\theta^2+\theta\big)\cdot\alpha^2-(\theta+1)^2\cdot\alpha\big)\cdot\big(\alpha^2+\big(\theta^2-3\big)\cdot\alpha-\theta^2-\theta\big)\Big)\cdot e^{-\frac{\left(-\sqrt{-1+\left(\theta^2+\theta\right)\cdot\alpha^2-(\theta+1)^2\cdot\alpha}+(\theta+1)\cdot(\alpha-1)\right)\cdot h\sim}{\alpha-1}} + \Big(\big(\big(\theta^2-1\big)\cdot\alpha-\theta^2-\theta-1\big)\cdot(\alpha-1)\cdot(2+\theta)\cdot\sqrt{-1+\big(\theta^2+\theta\big)\cdot\alpha^2-(\theta+1)^2\cdot\alpha} - \big(-1+\big(\theta^2+\theta\big)\cdot\alpha^2-(\theta+1)^2\cdot\alpha\big)\cdot\big(\alpha^2+\big(\theta^2-3\big)\cdot\alpha-\theta^2-\theta\big)\Big)\cdot e^{-\frac{h\sim\cdot\left(\sqrt{-1+\left(\theta^2+\theta\right)\cdot\alpha^2-(\theta+1)^2\cdot\alpha}+(\theta+1)\cdot(\alpha-1)\right)}{\alpha-1}} + 2\cdot\big(-1+\big(\theta^2+\theta\big)\cdot\alpha^2-(\theta+1)^2\cdot\alpha\big)\cdot(\alpha-1)^2\cdot(2+\theta)\Bigg)\Bigg)\Bigg/\Big(2\cdot\big(-1+\big(\theta^2+\theta\big)\cdot\alpha^2-(\theta+1)^2\cdot\alpha\big)\cdot\big(\theta^2+2\cdot\theta+2-(\theta+1)^2\cdot\alpha+(\theta+1)\cdot\alpha^2\big)\Big)$$

Consider a special case *{z=1, h=1}* for the convenience of constructing the surface of a double Laplace image, we introduce auxiliary functions

```
> g2:=unapply(2*(-1+(theta^2+theta)*alpha^2-
```

(theta+1)^2*alpha)*(theta^2+2*theta+2-(theta+1)^2*alpha+(theta+1)*alpha^2),theta,alpha);

$$g2 := (\theta, \alpha) \mapsto 2\cdot\left(-1 + \left(\theta^2 + \theta\right)\cdot\alpha^2 - (\theta+1)^2\cdot\alpha\right)\cdot\left(\theta^2 + 2\cdot\theta + 2 - (\theta+1)^2\cdot\alpha + (\theta+1)\cdot\alpha^2\right)$$

> g:=unapply(-1+(theta^2+theta)*alpha^2-(theta+1)^2*alpha,theta,alpha);

$$g := (\theta, \alpha) \mapsto -1 + \left(\theta^2 + \theta\right)\cdot\alpha^2 - (\theta+1)^2\cdot\alpha$$

> g3:=unapply(-((theta^2-1)*alpha-theta^2-theta-1)*(2+theta)*(alpha-1)*sqrt(g(theta,alpha)),theta,alpha);

$$g3 := (\theta, \alpha) \mapsto -\left(\left(\theta^2 - 1\right)\cdot\alpha - \theta^2 - \theta - 1\right)\cdot(\alpha - 1)\cdot(2+\theta)\cdot\sqrt{-1 + \left(\theta^2 + \theta\right)\cdot\alpha^2 - (\theta+1)^2\cdot\alpha}$$

> g4:=unapply(-g(theta,alpha)*(alpha^2+(theta^2-3)*alpha-theta^2-theta),theta,alpha);

$$g4 := (\theta, \alpha) \mapsto -\left(-1 + \left(\theta^2 + \theta\right)\cdot\alpha^2 - (\theta+1)^2\cdot\alpha\right)\cdot\left(\alpha^2 + \left(\theta^2 - 3\right)\cdot\alpha - \theta^2 - \theta\right)$$

> g5:=unapply(exp((sqrt(g(theta,alpha))+(theta+1)*(1-alpha))/(alpha-1)),theta,alpha);

$$g5 := (\theta, \alpha) \mapsto e^{\frac{\sqrt{-1 + \left(\theta^2+\theta\right)\cdot\alpha^2 - (\theta+1)^2\cdot\alpha} + (\theta+1)\cdot(1-\alpha)}{\alpha - 1}}$$

> g6:=unapply(exp((-sqrt(g(theta,alpha))-(theta+1)*(alpha-1))/(alpha-1)),theta,alpha);

$$g6 := (\theta, \alpha) \mapsto e^{\frac{-\sqrt{-1 + \left(\theta^2+\theta\right)\cdot\alpha^2 - (\theta+1)^2\cdot\alpha} - (\theta+1)\cdot(\alpha-1)}{\alpha - 1}}$$

> g1:=unapply((-1+(theta^2+theta)*alpha^2-(theta+1)^2*alpha)*(alpha-1)^2*exp(-alpha)*(theta+2)/((-1+(theta^2+theta)*alpha^2-(theta+1)^2*alpha)*((theta+1)*alpha^2-(theta+1)^2*alpha+theta^2+2*theta+2)),theta,alpha);

$$g1 := (\theta, \alpha) \mapsto \frac{(\alpha-1)^2\cdot e^{-\alpha}\cdot(2+\theta)}{\theta^2 + 2\cdot\theta + 2 - (\theta+1)^2\cdot\alpha + (\theta+1)\cdot\alpha^2}$$

> Kzhs:=unapply(((g3(theta,alpha)+g4(theta,alpha))*g5(theta,alpha)
+(-g3(theta,alpha)+g4(theta,alpha))*g6(theta,alpha))*
exp(-alpha)/g2(theta,alpha)+g1(theta,alpha),theta,alpha);

$$Kzhs := (\theta, \alpha) \mapsto \Bigg(\Bigg(\Big(-\big((\theta^2-1)\cdot\alpha-\theta^2-\theta-1\big)\cdot(\alpha-1)\cdot(2+\theta)\cdot\sqrt{-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha}-\big(-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha\big)\cdot\big(\alpha^2+(\theta^2-3)\cdot\alpha-\theta^2-\theta\big)\Big)\cdot e^{\frac{\sqrt{-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha}+(\theta+1)\cdot(1-\alpha)}{\alpha-1}}+\Big(\big((\theta^2-1)\cdot\alpha-\theta^2-\theta-1\big)\cdot(\alpha-1)\cdot(2+\theta)\cdot\sqrt{-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha}-\big(-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha\big)\cdot\big(\alpha^2+(\theta^2-3)\cdot\alpha-\theta^2-\theta\big)\Big)\cdot e^{\frac{-\sqrt{-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha}-(\theta+1)\cdot(\alpha-1)}{\alpha-1}}\Bigg)\cdot e^{-\alpha}\Bigg)\Big/\Big(2\cdot\big(-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha\big)\cdot\big(\theta^2+2\cdot\theta+2-(\theta+1)^2\cdot\alpha+(\theta+1)\cdot\alpha^2\big)\Big)+\frac{(\alpha-1)^2\cdot e^{-\alpha}\cdot(2+\theta)}{\theta^2+2\cdot\theta+2-(\theta+1)^2\cdot\alpha+(\theta+1)\cdot\alpha^2}$$

> plot3d(Kzhs(theta,alpha),theta=1..5,alpha=1..5,title=
"PART OF THE KZHS(theta,alpha) SURFACE ABOVE THE AREA
{1<=theta<=5,1<=alpha<=5}");

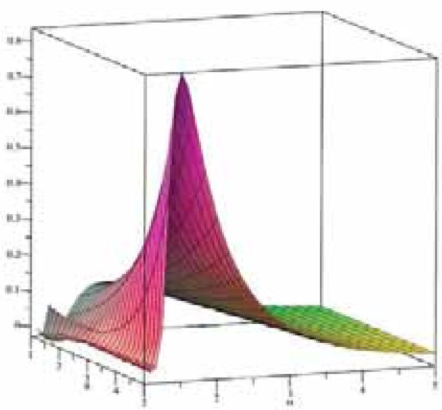

Fig. 1. We find one - dimensional inverse transformations
in t and in x for the g1 - component of the double Laplace image

> **g1t:=unapply(invlaplace(g1(theta,alpha),theta,t),t,alpha);**

$$g1t := (t\sim, \alpha\sim) \mapsto -\frac{1}{\sqrt{\alpha\sim^4 + 4\cdot\alpha\sim - 4}} \left(e^{\frac{\alpha\sim^2 \cdot t\sim - 2\cdot\alpha\sim^2 - 2\cdot\alpha\sim\cdot t\sim + 2\cdot\alpha\sim + 2\cdot t\sim}{2\cdot(\alpha\sim - 1)}} \cdot \sinh\left(\frac{t\sim\cdot\sqrt{\alpha\sim^4 + 4\cdot\alpha\sim - 4}}{2\cdot(\alpha\sim - 1)}\right)\cdot\left(\alpha\sim^2 + 2\cdot\alpha\sim - 2\right)\cdot(\alpha\sim - 1) \right)$$

$$- \cosh\left(\frac{t\sim\cdot\sqrt{\alpha\sim^4 + 4\cdot\alpha\sim - 4}}{2\cdot(\alpha\sim - 1)}\right)\cdot e^{-\alpha\sim}\cdot e^{\frac{(\alpha\sim^2 - 2\cdot\alpha\sim + 2)\cdot t\sim}{2\cdot(\alpha\sim - 1)}}\cdot(\alpha\sim - 1)$$

> **g1x:=unapply(invlaplace(g1(theta,alpha),alpha,x),x,theta);**

$$g1x := (x\sim, \theta) \mapsto \mathrm{Heaviside}(x\sim - 1)\cdot e^{\frac{(\theta+1)\cdot(x\sim - 1)}{2}} \cdot \left(\frac{2\cdot\sinh\left(\frac{(x\sim - 1)\cdot\sqrt{(\theta+1)\cdot(\theta^3 - \theta^2 - 5\cdot\theta - 7)}}{2\cdot(\theta+1)}\right)\cdot(\theta+2)}{\sqrt{(\theta+1)\cdot(\theta^3 - \theta^2 - 5\cdot\theta - 7)}} \right.$$

$$+ \frac{\cosh\left(\frac{(x\sim - 1)\cdot\sqrt{(\theta+1)\cdot(\theta^3 - \theta^2 - 5\cdot\theta - 7)}}{2\cdot(\theta+1)}\right)\cdot(\theta+2)\cdot(\theta-1)}{\theta+1}$$

$$+ \frac{1}{(\theta+1)^2\cdot(\theta^3 - \theta^2 - 5\cdot\theta - 7)}\left((\theta+2)\cdot(\theta^3 - \theta^2 - 5\cdot\theta - 5)\right.$$

$$\cdot\sqrt{(\theta+1)\cdot(\theta^3 - \theta^2 - 5\cdot\theta - 7)}$$

$$\left.\left.\cdot\sinh\left(\frac{(x\sim - 1)\cdot\sqrt{(\theta+1)\cdot(\theta^3 - \theta^2 - 5\cdot\theta - 7)}}{2\cdot(\theta+1)}\right)\right)\right)$$

> **evalf(Kzhs(5,5));**

0.01061314313

Solving the Cauchy problem with conditions *f(0)=0, D(f)(0)=0*
> **ics0:=f(0)=0,D(f)(0)=0;**

$$ics0 := f(0) = 0, \mathrm{D}(f)(0) = 0$$

> w0:=unapply(simplify(subs({mu=1,lambda=1},dsolve({eqss,ics0}, f(h)))),h,alpha,theta):

> Kzh0:=unapply(w0(h,alpha,theta)*exp(-alpha*z),z,h,theta,alpha);

$$Kzh0 := (z\sim, h\sim, \theta, \alpha) \mapsto f(h\sim)\cdot e^{-\alpha\cdot z\sim} = -\Bigg(e^{-\alpha\cdot z\sim}\cdot(\alpha-1)^2\cdot\Bigg(\Big((\theta+1)\cdot(\alpha-1)\cdot\sqrt{-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha}-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha\Big)\cdot e^{-\frac{\left(-\sqrt{-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha}+(\theta+1)\cdot(\alpha-1)\right)\cdot h\sim}{\alpha-1}} + \Big(-(\theta+1)\cdot(\alpha-1)\cdot\sqrt{-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha}-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha\Big)\cdot e^{-\frac{h\sim\cdot\left(\sqrt{-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha}+(\theta+1)\cdot(\alpha-1)\right)}{\alpha-1}} + 2 + (-2\cdot\theta^2-2\cdot\theta)\cdot\alpha^2 + 2\cdot(\theta+1)^2\cdot\alpha\Bigg)\cdot(2+\theta)\Bigg) \Big/ \Big(2\cdot(\theta^2+2\cdot\theta+2-(\theta+1)^2\cdot\alpha+(\theta+1)\cdot\alpha^2)\cdot(-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha)\Big)$$

> Kzh0s:=unapply(simplify(subs({z=1,h=1},Kzh0(z,h,theta,alpha))), theta,alpha):

> g10:=unapply((alpha-1)^2*(2+theta)/((theta+1)^2*(1-alpha)+(theta+1)*alpha^2+1)*exp(-alpha),theta,alpha);

$$g10 := (\theta, \alpha) \mapsto \frac{(\alpha-1)^2\cdot(2+\theta)\cdot e^{-\alpha}}{(\theta+1)^2\cdot(1-\alpha)+(\theta+1)\cdot\alpha^2+1}$$

> g20:=unapply(2*g(theta,alpha)*((theta+1)^2*(1-alpha)+(theta+1)*alpha^2+1),theta,alpha);

$$g20 := (\theta, \alpha) \mapsto 2\cdot(-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha)\cdot((\theta+1)^2\cdot(1-\alpha)+(\theta+1)\cdot\alpha^2+1)$$

> g30:=unapply((theta+1)*(alpha-1)*sqrt(g(theta,alpha)), theta,alpha);

$$g30 := (\theta, \alpha) \mapsto (\theta+1)\cdot(\alpha-1)\cdot\sqrt{-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha}$$

> Kzh0s:=unapply(g10(theta,alpha)-exp(-alpha)*(alpha-1)^2 *(theta+2)*((g30(theta,alpha)+g(theta,alpha))*g5(theta,alpha)+ (-g30(theta,alpha)+g(theta,alpha))*g6(theta,alpha))/g20(theta, alpha),theta,alpha);

$$Kzh0s := (\theta, \alpha) \mapsto \frac{(\alpha-1)^2\cdot(2+\theta)\cdot e^{-\alpha}}{(\theta+1)^2\cdot(1-\alpha)+(\theta+1)\cdot\alpha^2+1} - \Bigg(e^{-\alpha}\cdot(\alpha-1)^2\cdot(2+\theta) \cdot\Bigg(\Big((\theta+1)\cdot(\alpha-1)\cdot\sqrt{-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha} - 1 + (\theta^2+\theta)\cdot\alpha^2 - (\theta+1)^2\cdot\alpha \Big)\cdot e^{\frac{\sqrt{-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha}+(\theta+1)\cdot(1-\alpha)}{\alpha-1}} + \Big(-(\theta+1)\cdot(\alpha-1)\cdot\sqrt{-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha} - 1 + (\theta^2+\theta)\cdot\alpha^2 - (\theta+1)^2\cdot\alpha \Big)\cdot e^{\frac{-\sqrt{-1+(\theta^2+\theta)\cdot\alpha^2-(\theta+1)^2\cdot\alpha}-(\theta+1)\cdot(\alpha-1)}{\alpha-1}} \Bigg)\Bigg) \Bigg/ \Big(2\cdot\big(-1+(\theta^2+\theta)\cdot\alpha^2 - (\theta+1)^2\cdot\alpha\big)\cdot\big((\theta+1)^2\cdot(1-\alpha)+(\theta+1)\cdot\alpha^2+1\big)\Big)$$

> plot3d(Kzh0s(theta,alpha),theta=1..5,alpha=1..5);

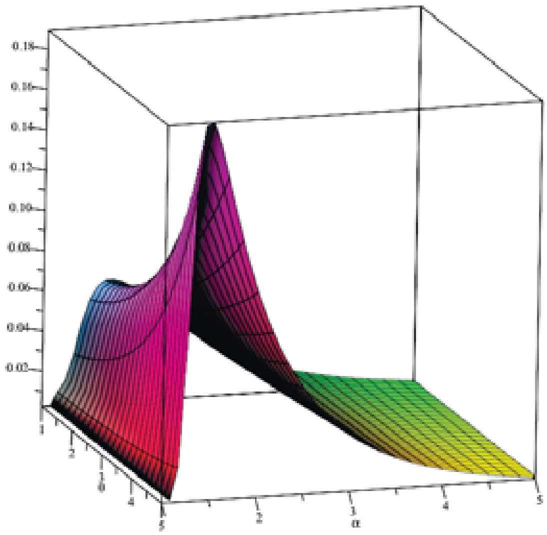

Fig. 2.

> evalf(Kzh0s(5,5));

0.0035525838

4. Conclusion

It has been proven in the paper that after the double integral transformation - the Laplace transform in time and the Laplace-Stieltjes transformation in phase, a function has been obtained that depends on the variables z and h and on the transformation parameters θ and α, which satisfies a certain fourth-order Mangeron equation in the variables z and h with respect to $\tilde{\tilde{R}}(\theta,\alpha|z,h)$. An analytical solution to this equation is found, a part of the solution surface is constructed, and a single inverse Laplace integral transformation is performed for a specific one component. The latter shows the possibility of applying a double inverse transformation, but this requires additional investigation of the resulting analytical expression for $\tilde{\tilde{R}}(\theta,\alpha|z,h)$.

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